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ESTIMATING MODEL FOR THE LINEAR ELECTRON DENSITY OF THE TRAIL CREATED BY A METEOROID

Introduction

The linear electron density of the trail (LEDTr) created by a meteoroid when passing the Earth's atmosphere at a given observation altitude is used to estimate the power of the received signal and predict the trail reflectivity in the process of its further transformation.

Unlike many existing formulas for calculating LEDTr and semi-empirical models obtained by fitting to the results of radar observations, the presented technique allows calculations to be carried out for explicitly specified parameters of the meteoroid body and atmosphere. The main physical processes occurring during the interaction of the meteoroid with atoms and molecules of the atmosphere were taken into account.

The obtained relation of LEDTr to altitude corresponds well to the luminosity curves of the meteor trail, and the value of the electron density at the ionization maximum corresponds to the known observational results.

Estimating LEDTr for a given (measured) reflection point provides the ability to calculate the amplitude-time characteristics (ATCH) of radio signals, which allows you to create a model image of the scattered signal.

Comparison of the ATCH received signal with the calculated model images allows make a reasonable assumption about the meteoroid characteristics that generated the ionized trail. Predictive estimates of the mass, density velocity and radiant of the meteoroid that formed the corresponding trail can be made, as well as assumptions about its chemical composition and the genetic relationship of the meteoroid to its parent body.

The purpose of the study is to create a model for estimating the linear electron density of the trail created by a meteoroid, taking into account the physicochemical properties of the meteoroid and atmosphere at the observation altitude, as well as constructing the relation of LEDTr to altitude.

The result of the study is a software-implemented model for estimating LEDTr at the observation altitude for explicitly specified physicochemical properties of the meteoric body and atmosphere.

Calculation methodology for the linear electron density of the trail

Free electrons are formed during the collision of ablated particles of a meteoric body (atoms and molecules) with atmospheric molecules, provided that the particles kinetic energy is sufficient to remove the electron, which is the least tightly bound to the nucleus. The number of collisions is proportional to the concentration of atmospheric particles and midsection of the head part of the meteoric body at the calculated altitude, and the number of electrons generated depends on the chemical composition of the meteoroid and the atmosphere at the observation altitude and is determined by the ionization coefficient.

Calculation of the linear electron density of the trail (LEDTr) involves the estimation of:

- midsection of the head part of the meteoric body at the observation altitude ($S_{MH}(hM)$);
- concentration of particles (numerical density) of the atmosphere at the observation altitude ($N_a(h_M)$);
- values of the integral ionization coefficient at the observation altitude ($\beta_{total}(hM)$).

These parameters depend on the physicochemical properties of the meteoroid (*PCPMet*) and the atmosphere at the altitude of observation of the ionized trail, and LEDTr can be calculated using the expression

$$q(v, m_m, h_M, PCPMet) = \beta_{total}(v, PCPMet) \times S_{MH}(v, m_m, h_M, PCPMet) \times N_a(h_M) \times l^{-1}, \quad (1)$$

where $\beta_{total}(v, PCPMet)$ is the integral estimate of the ionization coefficient, $N_a(h_M)$ is the numerical density of the atmosphere at the observation altitude h_M, and $l = \Delta h_M / \cos \gamma_m$ is the length of the path along which the electron density was calculated.

The most difficult task is to estimate the S_{MH} *(hM)* at the observation altitude. The value of the meteoroid's midsection at the entrance to the meteor zone without taking into account fragmentation and differential ablation is determined by the expression

$$S_{m0} = A_0 \left(\frac{m_{m0}}{\rho_m}\right)^{\frac{2}{3}}, \quad (2)$$

where A_0 is the shape coefficient, mm0 and ρm are the initial mass and density of the meteoroid, respectively. It is easy to show that for a ball $A_0 \approx 1.2$, and for a cube $A_0 = 1$.

The midsection S_{mM} at the observation point is determined by its residual mass m_{mM} in and expression (2) is converted to the form

$$S_{mM} = A_0 \left(\frac{m_{mM}}{\rho_m}\right)^{\frac{2}{3}}. \quad (3)$$

Particles that have evaporated within a given interval $\Delta l = \Delta h / \cos \gamma$ of the motion trajectory form a vapor "cushion" in front of the meteoroid, which leads to a change in the midsection of the meteoroid. In this case, the midsection of the head part at a altitude h_M can be represented by the expression

$$S_{HM}(h_M) = A_0 \left(\frac{m_{mM}}{\rho_m}\right)^{\frac{2}{3}} + N_{evep}(h_M) \cdot \sigma_{mav}, \quad (4)$$

where $\sigma_{m\,av}$ is the average cross–section for the collision of evaporated particles and N_{ever} (h_M) is the number of particles evaporated at altitude h_M. The number of evaporated particles can be represented by the expression

$$N_{ever}(h_M) = \frac{\Delta m_m(h)}{\mu_{mav}} = \frac{[m_m(h_M + \Delta h) - m_m(h_M)] \cdot N_A}{M_{mmol}} = \frac{\Delta m_m(h_M) \cdot N_A}{M_{mmol}} \quad (5)$$

where Δ_{mm} (h_M) is the mass loss at altitude h_M when it changes by Δh , $\mu_{m\,ev}$ is the average absolute mass of evaporated particles, which is defined as the ratio of the molar mass $M_{m\,mol}$ to the Avogadro number $N_A = 6.02214082 \cdot 10^{23}$ mole^{-1}. The molar mass of any substance, expressed in grams per mole, is numerically equal to the mass of the molecule of this substance, expressed in atomic mass unit, and can be found from the periodic table. Neglecting the mass defect, the relative atomic mass of a molecule is equal to the sum of the relative atomic masses of its constituent elements. It should be noted that if Δ_{mm} (h_M) was calculated with the unit of kg (SI system), the molar mass should be converted to kg/mole – i.e. divide by 1000.

Based on expressions (4) and (5), the midsection S_{MH} (h_M) at the observation point can be represented by the expression

$$S_{HM}(h_M) = A_0 \left(\frac{m_{mM}}{\rho_m}\right)^{\frac{2}{3}} + A_1 \frac{\Delta m_m(h) \cdot N_A}{M_{mmol}} \cdot \sigma_{mav}, \quad (6)$$

where A_1 is the coefficient correcting the increase in the midsection S_{MH} due to evaporation, and $\sigma_{m\,av}$ is the average value of the cross–section of the molecule (atom) of the meteoroid, which depends on its chemical composition.

The effective cross section of a molecule (atom) can be found by sequentially calculating the volume of a mole of a substance *X*, the volume of one molecule of a substance V(X1), and then the minimum distance d at which the centers of two molecules approach each other during a collision (the effective diameter of the molecule).

The mole volume of a substance X can be estimated by calculating its molar mass and dividing it by the density of this substance

$$V(X_{mole}) = \frac{M(X_{mole})}{\rho(X)}. \quad (7)$$

The molar mass of a substance $M(X_{mole})$ in kg/mol is numerically equal to the relative molecular weight of the given substance $\mu_m(X)$ divided by 1000, and its density $\rho(X)$ is selected from the appropriate reference books. An approximate estimate of the volume of one molecule of a substance $V(X_1)$ can be performed, assuming that the molecules are located as close as possible to each other and dividing $V(X_{mole})$ by the Avogadro number N_A

$$V(X_{mole}) = \frac{M(X_{mole})}{\rho(X)}. \quad (8)$$

In this case, the diameter of the molecule d_X is equal to

$$d_{X1} = \sqrt[3]{\frac{6V(X_1)}{\pi}}, \quad (9)$$

and the effective cross–section of a molecule (atom) of a meteoroid σ_{X1} is defined by the expression

$$\sigma_{X1} = \pi d_{X1}^2 = \frac{\pi}{100} \cdot \left[\frac{6\mu_m(X)}{\pi\rho(X)\cdot N_A}\right]^{2/3}, \quad (10)$$

and the average value of the cross section of evaporated particles $\sigma_{X1\ av}$ is equal to

$$\sigma_{X1ev} = \sum_{j=1}^{L} p_j\, \sigma_{X1j} = \sum_{j=1}^{L} p_j \frac{\pi}{100} \cdot \left[\frac{6\mu_m(X_j)}{\pi\rho(X_j)\cdot N_A}\right]^{2/3}, \quad (11)$$

where p_j is the fraction of the corresponding evaporated substance.

Based on expressions (5) and (10), the midsection $S_{MH}(h_M)$ at the observation point can be represented by the expression

$$S_{MH}(h_M) = A_0 \left(\frac{m_{m\mathrm{M}}(h_M)}{\rho_m}\right)^{\frac{2}{3}} + A_1 \frac{\Delta m_m(h)\cdot N_A}{M_{mmolav}} \cdot \sigma_{mev}, \quad (12)$$

where $M_{mmolav} = \sum_{j=1}^{L} p_j\, M_{mj}$.

To implement midsection calculations based on expression (11), it is necessary to obtain the relation of the meteoroid mass to the observation point altitude h_M, which will allow one to estimate the amount of evaporated matter at a given interval Δh. To do this, we will use the basic equations of the physical theory of meteors [1].

The first basic equation of the physical theory of meteors is based on the assumption that the loss of momentum by a meteoroid $m_m\ dv$ is proportional to the momentum of the oncoming air flow:

$$m_m \frac{dv}{dt} = -\Gamma S \rho_a v^2, \quad (13)$$

where m_m is the mass of the meteoroid, v is the velocity, S is the area of the frontal section of the meteoroid (midsection), ρ_a is the density of the atmosphere, Γ is the resistance coefficient (the fraction of the momentum of the incident atoms and atmospheric molecules, which is converted into the deceleration of the body).

The second basic equation of the physical theory of meteors describes the loss of mass of a meteoroid body, provided that all the energy is spent on ablation (evaporation or melting and blowing off the molten film):

$$\frac{dm_m}{dt} = -\Lambda S \rho_a \frac{v^3}{2Q}, \quad (14)$$

where Q is the specific heat of evaporation of the meteoroid substance (J/kg), and Λ is the fraction of the kinetic energy of the oncoming flow of molecules spent on ablation during the time dt.

We relate the change in time and the change in altitude using the relation [2]

$$dt = -\frac{dz}{v\cos\gamma}. \quad (15)$$

Then expressions (13) and (14) are transformed to the form

$$m_m \frac{dv}{dz} = -\frac{\Gamma}{Q_m \cos\gamma} S_m \rho_a v, \quad (16)$$

$$\frac{dm_m}{dz} = -\frac{\Lambda}{\cos\gamma} S_m \rho_a v^2. \quad (17)$$

Dividing (17) by (16) we obtain a relation connecting the mass m_m and velocity v of the meteoroid

$$\frac{1}{m_m} dm_m = \frac{\Lambda}{2Q_m\Gamma} v dv. \quad (18)$$

Integrating the left side of (18) in the range from the value of the initial mass of the meteoroid m_{m0} to its mass m_{mM} at the observation point, and the right side in the range from the value of the initial velocity of the meteoroid from v_{m0} to its value v_{mM} at the observation point, we obtain the equation

$$\int_{m_{m0}}^{m_{mM}} \frac{1}{m_m} dm_m = \frac{\Lambda}{2Q_m\Gamma} \int_{v_{m0}}^{v_{mM}} v\, dv. \quad (19)$$

As a result of integration, we obtain the value of the mass of the meteoroid at the observation point M depending on the velocity at this point v_{mM} for a given initial velocity v_{m0}

$$m_{mM} = m_{m0} \exp\left[-\frac{\Lambda}{2Q\Gamma} \frac{\left(v_{m0}^2 - v_{mM}^2\right)}{2}\right] \quad (20)$$

To calculate the meteoroid mass at an observation point, it is necessary to estimate its velocity at this point, which depends on the midsection of the meteoroid and the density of the atmosphere. In the first approximation, which is usually sufficient for many purposes of meteor astronomy, from the equation of state and the equation of hydrostatic equilibrium of the atmosphere, we obtain the exponential distribution of the atmospheric density $\rho_a\ (h)$ with respect to altitude h:

$$\rho_a(h) = \rho_b \exp\left[-\frac{g_b m_{ab}(h - h_b)}{RT_b}\right], \quad (21)$$

where ρ_b, g_b и T_b – atmospheric density, gravitational acceleration and absolute temperature at the h_b selected (base) altitude, m_{ab} – average molecular mass (kg/kmole) of the atmosphere at base altitude, and R = 8314.46 (J/kmole·˚K) is the universal gas constant.

If we choose h_b = 95000 m as the base altitude and use data from [3], formula (21) is converted to the form

$$\rho_a(h) = 1{,}4051 \cdot 10^{-6} \exp[-0{,}17768 \cdot 10^{-3}(h - 95000)]. \quad (22)$$

As a model for estimating the distribution of atmospheric density, the relation can be used

$$\rho_a(h) = 1{,}405 \cdot 10^{-6} \exp\left[-\frac{(h-95)}{H_a}\right], \quad (23)$$

where H_a= 46,8273 – 0,95h + 0,0055 h^2 – reduced altitude of the atmosphere (here h and H_a are measured in kilometers), as well as other models presented in [4].

In this study [5], when calculating the meteoroid velocity, it was assumed that its mass m_m and midsection S_m did not change when moving with an initial velocity v_{m0} in a medium with a density ρ_m, and the force of gravity was negligibly small compared to the force of resistance to motion. This made it possible to obtain an approximate estimate of the meteoroid velocity at the observation point represented by the expression

$$v_{mM}(h) = v_{m0}\, exp(-\rho*/2), \tag{24}$$

where $\rho* = \rho_a(h)/\rho_\beta(h)$, $\rho_\beta(h)=\beta \cos\gamma /h$, $\beta = m_m/S_m$ – ballistic coefficient, S_m – meteoroid midsection.

Taking into account the previously adopted notation, the expression (24) is transformed to the form

$$v_{mM}(h_{aM}, m_{mM}, \gamma_m) = v_{m0}\, exp\left[-\frac{\rho(h_{aM})h_{aM}S_{mM}C_D}{2m_{mM}\cos\gamma_m}\right], \tag{25}$$

where γ_m is the zenith angle of the meteoroid radiant; C_D is a dimensionless aerodynamic resistance coefficient, which, when a meteoroid moves in the range of meteor altitudes, can take values from 2 to 0.5 (depending on the form of the body and the Reynolds number for it). If at the altitude of observation the length free path of molecules is greater than the characteristic size of the body, the coefficient C_D can be taken equal to 2.

Expressing the midsection of the meteoroid S_{mM} terms of its mass m_{mM} and density ρ_m using formula (2) from expression (25) we obtain

$$v_{mM}(h_{aM}, m_{mM}, \gamma_m) = v_{m0}\, exp\left[-\frac{\rho_a(h_{aM})h_{aM}C_D A_0}{2(m_{mM}\rho_m^2)^{\frac{1}{3}}\cos\gamma_m}\right]. \tag{26}$$

The algorithm for calculating the values of m_{mM} and v_{mM} can be implemented based on calculation formulas (20) and (26) using the recurrent method, in which each next member of the sequence is calculated using the result of the calculations of velocity and mass at the previous step. This approach makes it possible to partially remove the restrictions regarding the requirement of constancy of the mass and midsection of the meteoroid during its movement.

To implement the recurrent method of calculating m_{mM} and v_{mM}, you must perform the following steps:

1. Calculate, using expression (26), the value of the velocity v_{mM} at a point close to the upper boundary of the meteor zone at a altitude $h_{max} - \Delta h$. At the same time, the meteoroid velocity at the entrance to the meteor zone v_{m0}, its initial mass m_{m0}, density ρ_m and radiant zenith angle γ_m are assumed to be known, and the altitude of the upper boundary of the meteor zone h_{max}, the coefficients C_D and A_0 are also considered given. The value of atmospheric density at altitude $h_{max} - \Delta h$ can be calculated using expression (22).

2. Calculate the meteoroid mass value at altitude $h_{max} - \Delta h$ using expression (20).

3. Calculate, using expression (26), the value of the velocity v_{mM} at the altitude $h_{max} - i\Delta h$, where $i = 2$. In this case, use the value of the meteoroid mass obtained at the previous step (at the altitude $h_{max} - \Delta h$).

4. Calculate the value of the meteoroid mass at the altitude $h_{max} - 2\Delta h$ using expression (20).

5. Calculate the value of the velocity v_{mM} at the altitude $h_{max} - i\Delta h$, where $i = 3$. In this case, use the value of the meteoroid mass obtained at the previous step (at the altitude $h_{max} - 2\Delta h$).

6. Repeat the calculations of v_{mM} and m_{mM} for $i = 4 \dots N$, where N is determined from the condition $(h_{max} - N\cdot\Delta h) \geq h_{min}$.

To calculate LEDTr using formula (1), it is necessary to determine the number density of the atmosphere Na(hM) (particle concentration) at a given altitude. This can be done on the basis of the reference data [3] or by using the approximate relationship

$$N_{\mathrm{a}}(h_{\mathrm{M}}) = \frac{\rho_a(h_{\mathrm{M}})N_A}{\mu_{aav}}, \tag{27}$$

where $\mu_{a\,ev}$ – average relative atomic mass of atmospheric particles (kg/kmol), which in the interval of 80 ... 130 km can be approximated by the expression

$$\mu_{aav}(h_{\mathrm{M}}) = \begin{cases} 28{,}964 & \text{when } 95km \geq h_{\mathrm{M}} \geq 80km \\ 28{,}964 - 0{,}14056(95 - h_{\mathrm{M}}) & \phantom{\text{when }} h_{\mathrm{M}} > 95km \end{cases} \tag{28}$$

For a complex chemical composition meteoroid, the integral estimate $\beta_{total}(v)$ is a linear combination of ionization coefficients, where each coefficient is taken with a weight p_j proportional to the fraction of atoms present in the composition of the meteoroid

$$\beta_{total}(v) = \sum_j p_j \beta_j(v). \tag{29}$$

The value of $\beta_j(v)$ depends on the velocity and ionization potential of the atoms that make up the meteoric body, which can be found in the corresponding reference books.

The works [1, 6–8] present an analysis of various methods for calculating the ionization coefficient, which assume the possibility of obtaining estimates of $\beta(v)$ under various assumptions regarding the physicochemical composition of meteoroids and the atmosphere. Average data for $\beta(v)$ of these elements are presented in Table 1 [1].

Table 1

Element	**Fraction, %**	**Meteoroid velocity *v*, km/s**		
		20	**40**	**70**
O	56,0	0,00016	0,043	0,554
Fe	11,4	0,068	0,595	3,41
Mg	15,4	0,020	0,151	0,81
Ca	0,9	0,082	0,619	1,98
Si	14,4	0,023	0,300	1,70
β (v)		**0,0154**	**0,170**	**1,12**

Based on these data in the study [1], an approximating calculation formula for $\beta(v)$ was obtained

$$\beta(v) = 5{,}4889 \times 10^{-7} \times v^{3{,}42}. \tag{30}$$

This formula can be used to perform approximate calculations of the ionization coefficient $\beta(v)$ for meteoroids of various origins.

For iron meteoroids in the study [7], it was proposed to calculate $\beta(v)$ using the formula

$$\beta_{Fe}(v) = 5{,}96 \times 10^{-6} \times v^{3{,}42}, \tag{31}$$

which is applicable for the velocity range of 20 km/s $\le v \le$ 45 km/s).

Based on expression (1), using relations (10), (11), (22), (20), (26), (28), (30) (or (31)) we obtain a formula that allows us to calculate LEDTr

$$q(v, m_m, h, \gamma) = \beta_{total}(v) \times \left[A_0 \left(\frac{m_{m\text{M}}}{\rho_m}\right)^{\frac{2}{3}} + A_1 \frac{\Delta m_m(h,v,\gamma)\cdot N_A}{\mu_{mev}} \sigma_{X1cp}\right] \times \left[\frac{\rho_a(h_\text{M}) N_A}{\mu_{aev}(h_\text{M})}\right] \times \frac{\cos\gamma}{\Delta h_\text{M}} \tag{32}$$

The average value of the mass of atoms $\mu_{m\ av}$ in a molecule of a meteoric substance can be determined based on its chemical formula. For example, a meteoroid like the Chelyabinsk meteorite mainly consists of silicates: olivines (($Mg,Fe)_2[SiO_4]$ and pyroxenes $(Mg,Fe)_2[Si2O_6]$. An olivine molecule contains 2 Fe atoms = 55.847 amu, 2 Mg atoms = 24.305 amu, 1 Si atom = 28.085 amu, and 4 atoms O = 15.9994 amu (we take the atomic mass from the periodic table). Determine its molar mass M_{oliv}. = 252.3866 amu and the proportion of each atom in the composition of the olivine molecule. After this, it is easy to show that the average mass of atoms in an olivine molecule is $\mu_{m\ av}$ = 36.5785 amu. Density ρ_m for olivine is usually close to 3300 kg/m^3.

Simulation results

Table 2 represents the initial data for calculating LEDTr created by iron meteoroids and meteoroids with a chondritic structure, and Table 3 shows the calculation algorithm using formula (32) and formulas used for calculation.

Table 2

No.	Parameters	Designation, dimension	Value
1	The meteoroid velocity upon entering the meteor zone	v_{m0}, *m/s*	40000
2	Zenith angle of the meteoroid	γ, *degree*	60
3	Initial meteoroid mass	m_{m0}, *kg*	0,001
4	Upper limit of the meteor zone	$h_{M\,max}$, *m*	130000
5	Lower boundary of the meteor zone	$h_{M\,min}$, *m*	80000
6	Mass density of iron meteoroid	$p_{m\,Fe}$, kg/m^3	7874
7	Iron atom mass	$\mu_{m\,Fe}$, *amu*	55,8430
8	Mass density of olivine $(Mg,Fe)_2[SiO_4]$	$p_{\text{m олив}}$, kg/m^3	3300
9	Average atomic mass in an olivine molecule	$\mu_{m\,cp}$, *amu*	36,5785
10	Aerodynamic resistance coefficient	C_D	2
11	Shape factor	A_0	1,21
12	Correction factor	A_1	1
13	Fraction of the kinetic energy of the oncoming flow of molecules spent on ablation	Λ	1
14	Specific heat of vaporization of meteoroid matter	Q, $J/kg\ (m^2/s^2)$	$6{,}3\cdot10^6$
15	Atmospheric resistance coefficient, characterizing the fraction of momentum transmitted to the meteoroid by atmospheric particles	Γ	1

Table 3

No.	Actions Performed	Formulas for calculations
1	Set the values of observation altitudes h_M in the meteor zone with a step Δh_M. In this case, $h_{Mi} = h_{M\,max} - i\,\Delta h_M$, where the value of i varies from 0 to $N = (h_{M\,max} - h_{M\,min})/\Delta h_M$.	
2	Calculate the average value of the cross section of evaporated particles of the meteoric body $\sigma_{X1\,av}$	(10), (11)
3	Calculate the atmospheric density $\rho_a(h)$ at observation altitudes h_{Mi}.	(22)
4	Calculate the velocity v_{mM} and mass m_{mM} of the meteoroid for the selected altitudes using the proposed iteration method. Take the initial mass and velocity of the meteoroid upon entering the meteor zone from Table 1.	(26), (20)
5	Calculate the meteoroid mass loss Δm_{mM} over the interval Δh in the area of the selected altitude h_{Mi}, subtracting the subsequent value from the previous mass value.	
6	Calculate the average relative atomic mass of atmospheric particles $\mu_{a\,ev}$	(28)
7	Calculate the value of the ionization coefficient $\beta_{total}(v)$for a meteoroid with a complex chemical composition.	(30) or (31) for iron
8	Calculate LEDTr using the proposed formula	(32)

The results of the calculation of LEDTr, performed using Microsoft Excel based on the proposed algorithm for iron meteoroids and meteoroids with a chondritic structure (close in composition to olivine) are presented in Figure 1. The initial data for calculations are taken from Table 2.

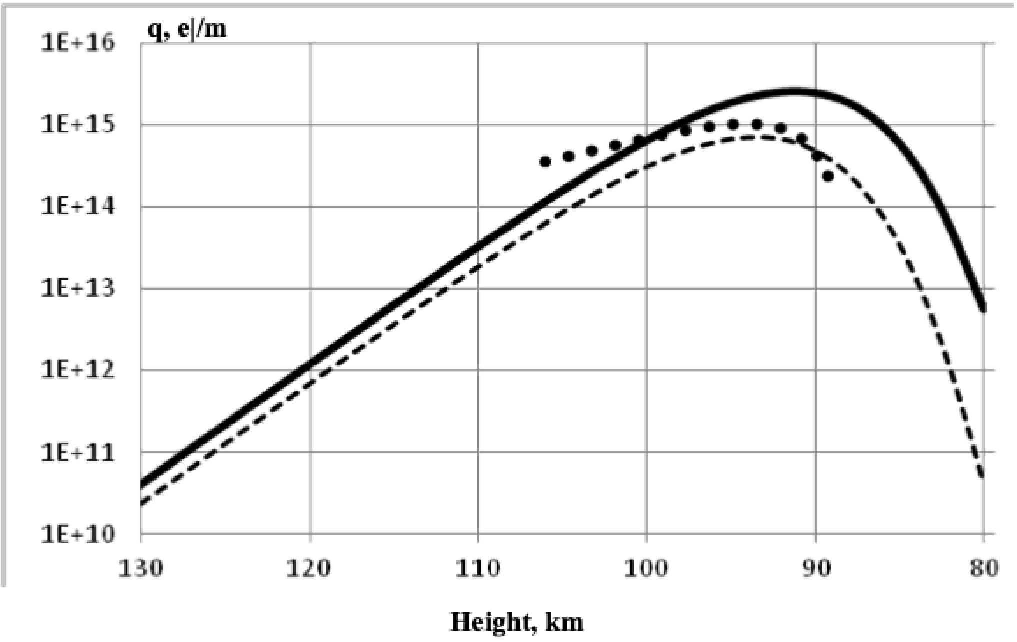

Fig. 1. Relation of LEDTr of an iron meteor body to the altitude of the observation point – solid line. The dotted line shows LEDTr for a meteoroid close in composition to the Chelyabinsk meteorite, and the dots show the result of the calculation using the semi-empirical formula (33)

In the study [9], based on previously obtained results [10 – 13], it was proposed to calculate the linear electron density (in electrons/m) using a semi-empirical formula, valid only in the region of maximum ionization and obtained as a result of fitting to the results of radar measurements

$$q(h_M) = 4{,}03 \times 10^{14} \cdot \frac{m(v_{m0}-8{,}15)^3}{H_M} \cdot cos\,\gamma \cdot Z(t), \qquad (33)$$

where h_M – altitude above the Earth's surface at the considered point on the trail;
v_{m0} – velocity (in km/s) of the meteoroid upon entering the meteor region;
m – mass (in kg) of the meteoroid upon entering the meteoroid region;
γ – zenith angle of the meteoroid;
$Z(t)$ – function defined by the expression

$$Z(t) = \begin{cases} \frac{9}{4} e^{-t}\left(1-\frac{1}{3}e^{-t}\right)^2 & \text{when} \quad -ln\,3 \le t \le 1.7 \\ 0 & \quad t \le -ln\,3\,, t \ge 1.7 \end{cases} \qquad (34)$$

where the relative altitude t is defined as

$$t = \frac{h_M - h_{max}}{H_M}; \qquad (35)$$

H_M – reduced altitude of the atmosphere (in km), which is calculated by the formula

$$H_M(h_M) = 6{,}4 + 0{,}09(h_M - 95); \qquad (36)$$

h_{max} is the altitude of the maximum ionization (in km), determined by the empirical relation

$$h_{max} = 47{,}4 + 12{,}76 * ln\,v_{m0}. \qquad (37)$$

The results of calculations using formulas (33) – (37) are represented in Figure 1 by a dotted line. They show a fairly good agreement with the corresponding calculations of LEDTr calculated using the proposed method.

Discussion

When developing the represented model for estimating the linear electron density of the trail (LEDTr), the main physical processes occurring during the interaction of a meteoroid with atoms and molecules of the atmosphere were taken into account. The calculation of LEDTr includes an assessment of midsection of the head part of the meteoric body, the concentration of particles (numerical density) of the atmosphere and the value of the integral ionization coefficient at the observation altitude.

When developing this model, well-known models for estimating the density of the atmosphere at a given altitude and the ionization coefficient, which depends on the chemical composition of the meteoroid, were used. The corresponding calculation formulas were obtained, and a software-implemented calculation algorithm was presented. It is important to note that when estimating the mass loss of the meteoroid, a recurrent method was used to calculate the mass and velocity as its altitude changed. This method made it possible to take into account changes in the mass and midsection of the meteoroid during its movement.

Unlike many existing formulas for calculating LEDTr and semi-empirical models obtained by fitting to the results of radar observations, our technique allows us to carry out calculations for explicitly specified parameters of the meteoric body and atmosphere. The obtained relation of LEDTr to altitude corresponds well to the luminosity curves of the meteor trail, and the value of the electron density at the ionization maximum corresponds to the known results.

Calculations of LEDTr at a given (measured) altitude, performed using the proposed methodology, make it possible to estimate the power of the received signal and predict the reflectivity of the trail in the process of its transformation. This, in turn, makes it possible to calculate amplitude-time characteristics and create a model image of a signal scattered at a given altitude.

The presented model assumes that the meteoroid is a single body, and the main mechanism of ablation is evaporation without crushing or changing the meteoroid shape. However, the model can be adapted for the case of fragmentation of a meteoroid at a given altitude. To do this, it is necessary to calculate LEDTr for each fragment according to the described method, and then use the principle of superposition, according to which the resulting effect of several independent events can be represented by the sum of the effects caused by each event separately. With quasi-continuous crushing of meteoroids, adjustment to the results of radar measurements is necessary. The proposed calculation formula provides a correction factor that allows you to change the value of the meteor body's midsection. This coefficient may differ for meteoroids of different origins.

It should be noted that the presented model for estimating LEDTr can be modified, since it did not take into account the decrease in electron density due to the recombination of positive ions with electrons, the attachment of free electrons to oxygen molecules, as well as ion reactions with ozone. The known estimates of atmospheric density and ionization coefficient used in developing the model can be specified. In this case, the calculation algorithm remains unchanged.

Conclusions

1. The model proposed for estimating the linear electron density of the trail (LEDTr) created by a meteoroid at a given (measured) altitude allows calculations to be carried out for explicitly specified parameters of the meteoroid and atmosphere.

2. The presented methodology and a software-implemented calculation algorithm make it possible to obtain relation of LEDTr to the observation altitude, which qualitatively coincides with the results of radar observations and the luminosity curve of the meteor trail.

3. LEDTr calculations using the proposed method make it possible to estimate the power of the received signal, calculate its amplitude-time characteristics and create a model image of the signal scattered at a given altitude.

4. The known estimates of atmospheric density and ionization coefficient used in developing the model can be refined. In this case, the calculation algorithm remains unchanged.

5. The presented model for estimating LEDTr can be modified, since it does not take into account the decrease in electron density due to the recombination of positive ions with electrons, the attachment of free electrons to oxygen molecules, as well as ion reactions with ozone.

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