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THEORETICAL INVESTIGATION OF INJECTION-LOCKED DIFFERENTIAL OSCILLATOR

Introduction

Application of ingection-locked differential LC oscillator began at the beginning of this century, simultaneously with the advent of an autonomous differential LC oscillator. Currently, synchronized differential LC oscillators are an essential part of not only communication systems, but also many automation and measuring devices. The synchronization mode allows significantly expanding the functional posibility of oscillators. Practical application was found not only for separate fundamentally synchronized oscillators but also for the frequency division and multiplication modes, and more complex devices including several coupled oscillators [1 – 12].

The study of the synchronized differential oscillators is a difficult problem, and a satisfactory method for performing this kind of work has not yet been developed. The lack of an adequate mathematical model, permitting the application of rigorous mathematical methods of the nonlinear theory of electrical oscillations, was the reason for using a simplified approach. Usually it was reduced to a transition to a simpler single- circuit LC oscillator, being equivalent to a differential one. However, no justification for such a transition was provided.

The circuit diagram of a single-circuit LC-oscillator was shown as a parallel LC circuit connected to a bipolar active element, being a current generator. At first it was considered that the current had the form of square wave, according to the operating mode of the differential oscillator transistors, which are represented as ideal switches, distributing the direct current I_0 (the bias current, which is considered known) between differential oscillator branches. The current of this source varied from $+I_0$ to $-I_0$, that was an idealization, but made it possible to determine the amplitude of the first harmonic and, as a consequence, the amplitude of the oscillator signal. Such a range of current variation provided an approximate equality of the signals amplitudes of the differential oscillator and its equivalent.

Many authors proposed a mathematical model of the equivalent oscillator in the form of nonlinear differential equations, representing the nonlinear characteristic of the active element by a piecewise constant function I_0 sgn(), where the argument was the oscillator signal. Instead of such an expression for the nonlinear characteristic, the first harmonic current component of the active element could also be used directly, which was easily determined. Such representations were used in many papers, for example [1, 2, 5].

Improving the differential oscillator study results is also associated with the equivalent oscillator, as in the previous cases, but with the representation of the nonlinear characteristic of the active element by a polynomial of the third degree [3 – 9, 12]. In the works, it was indicated without details, that this nonlinear characteristic was determined by modeling the differential oscillator in the Spice simulator and was quite accurately approximated by the above mentioned polynomial. However, the method of experimental determination was not presented, despite the fact that this nonlinear characteristic had a very specific form. Besides, a relation of the the equivalent oscillator active element nonlinear characteristic and the nonlinear characteristics of the two amplifying elements of the differential oscillator has not been established.

The above problems, solved for an autonomous differential oscillator [13], are partially used in this paper.

The analysis of publications devoted to synchronized differential oscillators leads to a conclusion that the problem of a rigorous theoretical study of such devices has not been satisfactorily resolved.

Thus, the purpose of the paper is to study the fundamentally injected differential LC -oscillator by rigorous mathematical methods of the nonlinear theory of electrical oscillations.

Synchronized oscillator equation

Let us consider the circuit diagram of the oscillator shown in Fig. 1, which consists of two connected identical single-circuit LC- oscillators. The equations describing the operation of the differential oscillator are derived under the following commonly used assumptions: u_0=const, I_0=const, the transistors are identical and are inertialess amplifying elements, the influence of their input and output resistances can be neglected. The nonlinear characteristics of transistors are approximated by a polynomial of the third degree $i = a_0 + a_1 u_y + a_2 u_y^2 + a_3 u_y^3$, where u_y is the control voltage. For transistor M_1 $u_y = u_{g1} = V_{DD} - u_0 + e$ and for transistor M_2 $u_y = u_{g2} = V_{DD} - u_o + u$, where u_{g1} is the gate voltage of transistor M_1 and u_{g2} is the voltage at the transistor gate M_2, u *and* e are variable voltage components.

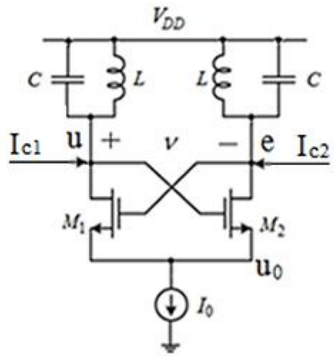

Fig. 1 Circuit diagram of the oscillator

Using Kirchhoff's laws, the system of nonlinear differential equations of the differential oscillator can be represented as:

$$\begin{aligned} &\frac{d^2u}{dt^2} + \frac{1}{C}\frac{d}{dt}\left(\frac{u}{R} - i_1\right) + \omega_0^2 u = \frac{1}{C}\frac{d}{dt} i_{c1}, \\ &\frac{d^2e}{dt^2} + \frac{1}{C}\frac{d}{dt}\left(\frac{e}{R} - i_2\right) + \omega_0^2 e = \frac{1}{C}\frac{d}{dt} i_{c2}, \end{aligned} \quad (1)$$

where i_1 and i_2 are the currents flowing through the nonlinear elements M_1 and M_2, R, ω_0 are the resonant resistance and frequency of the resonant circuits, $i_{c1} = I_{c1} cos * (\omega_c t + \varphi_1)$, $i_{c2} = I_{c2} cos * (\omega_c t + \varphi_2)$, $I_{c1} = I_{c2} = const$, $\omega_c \approx \omega_0$.

In accordance with the algorithm of the oscillator operation, the variable components of the voltages on the transistors drains are antiphase, i.e. $e = -u$, and the information parameter is their difference $\nu = u - e$. Subtracting the second equation from the first equation of the system (1), we obtain the differential equation for the transition to an equivalent oscillator

$$\frac{\mathrm{d}^2\nu}{\mathrm{d}t^2} + \frac{1}{\mathrm{C}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\nu}{\mathrm{R}} - \left(i_{1(u_{g1})} - i_{2(u_{g2})}\right)\right) + \omega_0^2 \nu = \frac{1}{\mathrm{C}}\frac{\mathrm{d}}{\mathrm{d}t}(i_{\mathrm{c}1} - i_{\mathrm{c}2}) \quad (2)$$

where $i_{1(u_{g1})} - i_{2(u_{g2})} = i$ is the current of the equivalent oscillator active element. The problem is to find a function $i_{(\nu)}$, so that $i_{(\nu)}$=$i_{1(u_{g1})}$−$i_{2(u_{g2})}$.

After solving this problem, the equation (2) will describe the operation of the oscillator which is an equivalent to the differential one, but having one resonant circuit and one amplifying element. Moreover, this circuit is identical to the resonant circuits of the differential oscillator, and the nonlinear characteristic of the active element is different from the nonlinear characteristics of the differential oscillator amplifying elements and depends on them. Thus, equation (2) turns into the Van Der Pol equation and can be studied by rigorous mathematical methods of the theory of nonlinear electrical oscillations.

Nonlinear characteristic of the equivalent oscillator amplifying element

Let us start with the nonlinear characteristics of the differential oscillator amplifying elements being identical and forming nonlinear characteristic of the equivalent oscillator hypothetical amplifying element. Due to this identity, as well as for simplicity and clarity, we will proceed from the nonlinear characteristic of one amplifying element, shown in Fig. 2. However, we take into account that the parameters u and e are antiphase. First of all, according to [13], it is necessary to determine positions of the differential oscillator operating points.

Obviously, $\nu = 0$, only if $u = 0$ and $e = 0$. In this case, the voltages at the gates of the transistors are equal to the voltages on their drains and are described by the expression $u_{g10} = u_{g20} = V_{DD} - u_0$. Then, the transistor currents are also the same and equal to $I_0/2$. Consequently, the operating points of the amplifying elements of the differential oscillator are described by the parameters $u_{\text{pr}} = u_{g10} = u_{g20} = V_{DD} - u_0$ and $I_{\text{pr}} = I_0/2$. In Figure 2, the operating point is indicated by the symbol u_{pr}.

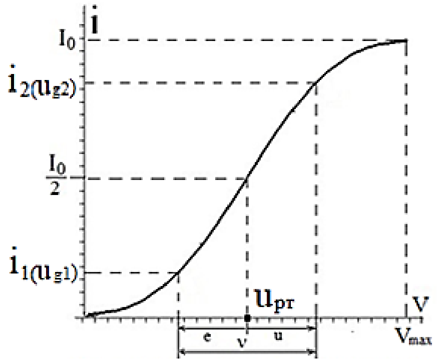

Fig. 2 Nonlinear characteristic of the amplifying element

Next, in Fig. 2 we mark points $u_{g1} = V_{DD} - u_0 - e$ and $u_{g2} = V_{DD} - u_o + u$, where $u_{g1} < u_{g2}$, and e and u are absolute values, then we find the corresponding currents $i_{1(u_{g1})}$ and $i_{2(u_{g2})}$ and determine their difference $i_{1(u_{g1})} - i_{2(u_{g2})} < 0$. This difference corresponds to a certain parameter $\nu = u_{\text{M1}} - u_{\text{M2}} = u_{g2} - u_{g1} = u + e > 0$, which allows writing $i_{1(\text{u}_{\text{g1}})} - i_{2(\text{u}_{\text{g2}})} = i_{(\nu)}$. These values permit to find the position of the point corresponding to the nonlinear characteristic of the equivalent oscillator amplifying element. Thus, for $u_{g2} - u_{g1} > 0$, $i_{(\nu)} = i_{1(\text{u}_{\text{g1}})} - i_{2(\text{u}_{\text{g2}})} < 0$. This case is shown in Fig. 2. So, the negative values of the difference $i_{1(u_{g1})} - i_{2(u_{g2})}$ correspond to the positive value of the parameter ν.

It is easy to see that a similar dependence can be obtained for the case $u_{g2} - u_{g1} < 0$ i.e. $\nu < 0$, where $i_{(\nu)} = i_{1(u_{g1})} - i_{2(u_{g2})} > 0$. This case will be shown in Fig. 2, if we swap the symbols e and u, as well as the symbols $i_{1(u_{g1})}$ and $i_{2(u_{g2})}$.

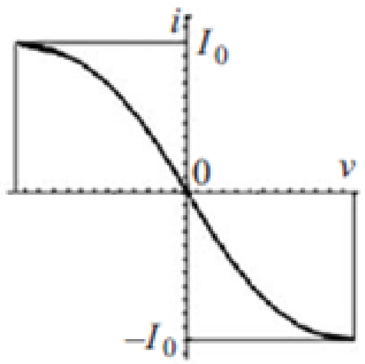

Fig. 3 Nonlinear characteristic of the equivalent oscillator amplifying element

Acting in accordance with this algorithm one sets different values of the parameters u_{g1} and u_{g2}, determines current values $i_{1(u_{g1})}$ and $i_{2(u_{g2})}$, as well as the difference $i_{1(\text{u}_{\text{g1}})} - i_{2(\text{u}_{\text{g2}})} = i_{(\nu)}$ and corresponding values of the parameter ν. As a result the function $i = f(\nu)$ is obtained, which describes the nonlinear characteristic of the active element. It is presented graphically in Fig. 3.

Obviously, this nonlinear characteristics belongs to an electronic device with negative differential resistance, and the equivalent oscillator is an oscillator with internal positive feedback. This circumstance will be taken into account in the differential equation (2), if the sign “minus”, in front of the term containing the currents, is changed into the sign “plus”. Then, the differential equation of the equivalent oscillator takes the form:

$$\frac{\mathrm{d}^2\nu}{\mathrm{d}t^2} + \frac{1}{\mathrm{C}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\nu}{\mathrm{R}} + i_{(\nu)}\right) + \omega_0^2\nu = \frac{1}{\mathrm{C}}\frac{\mathrm{d}}{\mathrm{d}t}i_{\mathrm{c}}, \tag{3}$$

where $i_{\text{c}} = i_{\text{c1}} - i_{\text{c2}} = 2I_{c1}\sin\left(\frac{\varphi_1 - \varphi_2}{2}\right)\sin\left(\omega_c t + \frac{\varphi_1 + \varphi_2}{2}\right)$.

It is easy to see that the values of the synchronizing signals phases have a very strong effect on the amplitude of the equivalent oscillator synchronizing signal. Let us consider the case when the phase difference is equal 180^0. Then

$$i_{\text{c}} = i_{\text{c1}} - i_{\text{c2}} = 2I_{c1}\,cos(\omega_c t + \varphi_1) = I_c\,cos(\omega_c t + \varphi_1),$$

where $I_c = 2I_{c1}$.

The obtained nonlinear characteristic, shown in Fig. 3, is symmetric about the origin, and it is an odd function. This also means that the bias is zero, which simplifies the investigation. For research, this nonlinear characteristic is approximated by a polynomial of the third degree $i = -a_1 v + a_3 v^3$.

The equivalent oscillator mathematical model

Having an analytical expression describing the nonlinear characteristic of the amplifying element, the study is simplified because the equation is the Van der Pol equation with a positive feedback coefficient equal to one. The mathematical methods used in this case are rigorous and the methodology has been worked out and well tested. Then, the equation (3) can be represented in the form

$$\frac{d^2 v}{dt^2} - \varepsilon\omega_0 \frac{dv}{dt}(v + \gamma v^3) + \omega_0^2 v = R\delta\omega_0 \frac{\mathrm{d}}{\mathrm{dt}} i_{\mathrm{c}}, \tag{4}$$

where $\varepsilon = \delta\alpha$ is a small parameter, $\alpha = Ra_1 - 1$ is the regeneration coefficient, $\alpha_0' = -a_1 + 1/R$, $\gamma = a_3/\alpha_0'$, $\delta = 1/Q$, ω_0, R, Q are the resonant frequency of the oscillator circuit, its resistance and quality factor.

This differential equation in the most general form describes the processes in the oscillator and, to simplify the study, we present it in a dimensionless form. In the first approximation, we can consider that the oscillations are harmonic $v = A_0 \cos(\tau + \varphi_0)$ and find the oscillation amplitude of the autonomous oscillator in the steady state. Substituting this expression into the original equation (4) and taking into account only the components of the fundamental frequency, after simple transformations, we obtain:

$$A_0 = \sqrt{-4/(3\gamma)}$$

When choosing a method for solving equation (4), it is necessary to estimate the values of its terms. To do this we introduce dimensionless variables $\tau = \omega_c t$, $v_n = v/A_0 \le 1$, $I_{cn} = I_c/I_0 \ll 1$, where $I_0 = A_0/R$, $i_c = I_{cn}\, cos(\omega_c t + \varphi_1)$, and take into account that $\omega_c \approx \omega_0$. Then, it is possible to write:

$$\frac{d^2 v_n}{d\tau^2} - \varepsilon\left[\frac{d}{d\tau}\left(v_n - \frac{4}{3}v_n^3\right)\right] + \frac{\omega_0^2}{\omega_c^2} v_n = \delta \frac{\mathrm{d}}{\mathrm{dt}} i_{\mathrm{c}}, \tag{5}$$

Now, when the values of the variable v_n are known, it is easy to see that, for small values of the small parameter and the small synchronization signal, this equation describes the behavior of a weakly nonlinear system and its solution, as is known, can be represented in the form $v_n = A_{(\tau)} \cos(\tau + \varphi_{(\tau)})$. A rigorous, well-established methods, such as the slowly varying amplitude method or the averaging method, can be used to find the amplitude and phase of oscillations. However, the above mentioned methods lead to systems of nonlinear shortened differential equations, presenting significant difficulties, when solving. An approximate analytical method, satisfying the needs of practice in most cases, developed recently, is given in [14].

Thus, the study of the synchronized differential oscillator is reduced to the study of the synchronized Van der Pol oscillator.

Conclusion

The paper presents the study of the fundamentally injected differential LC- oscillator by rigorous methods of the nonlinear theory of electrical oscillations in the case of small values of the small parameter. A research methodology has been proposed, an adequate mathematical model has been obtained. It makes it possible to study the differential oscillator as easily as the Van Der Pol oscillator. This model allows studying small but important effects, such as fluctuations of the amplitude and phase of oscillations and other significant parameters. It can be useful when developing devices using differential oscillators.

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