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A.I. KOZAR, Dr. Sc. (Math. And Phys.)

RESONANT GRATING OF MAGNETODIELECTRIC RESONANCE NANOSPHERES

Introduction

The advancement of nanotechnologies is related with the use of metamaterials whose structure discreteness and associated resonance phenomena are of primary importance. Interaction between electromagnetic radiation and metacrystals at the resonance domain is complicated and insufficiently studied. Methods for electromagnetic simulation of the phenomena in crystal lattices metacrystals are developed in order to help studying them.

The investigations of electromagnetic waves scattering on a plane grating wherein both the structural electromagnetic interaction between grating scattering elements and scattering elements on their own have resonant properties, are of significant interest in practice.

The work is aimed to solve in the interconsistent statement the problem of electromagnetic waves scattering on a plane grating of identical small inhomogeneous isotropic resonant magnetodielectric spheres. In the given problem, a length of the scattered wave can be commensurable with constants of the grating, what enables one to take into account the influence of grating structural resonances of the electromagnetic interaction between spheres on internal resonances of spheres and their fine structure.

The solution is obtained on the basis of the second type Fredholm integral electromagnetics equations [1].

We shall use the results of solution of problems considered in [2, 3, 4].

Statement and solution of the problem

Consider a plane grating of nodes that is generated in Cartesian co-ordinates by the coordinate representation in the view [3]

$$\begin{aligned}
&\left(x_{p,s} = x_s,\, y_{p,t} = y_t\right)\\
&x_s = \left[s - 0{,}5\left\{(-1)^s - 1\right\}\right]d - (-1)^{s-1} x_{s=0} \quad (s=0, \pm 1, \pm 2, \ldots),\\
&y_t = \left[t - 0{,}5\left\{(-1)^t - 1\right\}\right]h - (-1)^{t-1} y_{t=0} \quad (t=0, \pm 1, \pm 2, \ldots),\\
&z_p = z_{p=0} = 0,
\end{aligned} \tag{1}$$

where values d, h are defined by the conditions $x = 0$, $x = d$; $y = 0$, $y = h$, whereas $x_{s=0}$, $y_{t=0}$, $z_{p=0}$ are the coordinates of a node giving birth to the grating and located within a domain (Fig. 1):

$$\begin{aligned}
&0 \le x_{s=0} \le d,\\
&0 \le y_{t=0} \le h,\\
&z_{p=0} = 0.
\end{aligned} \tag{2}$$

Co-ordinates x_s, y_t determine positions of nodes outside domain (2) and are functions of co-ordinates $x_{s=0}, y_{t=0}$. It is possible to introduce the time dependence in the coordinate representation, when considering coordinates $x_{s=0}, y_{t=0}$ as some functions of time. Each node confronts with numbers $c = (s,t)$. We shall designate a singled-out grating node as $c' = (s', t')$, whereas a node within domain (2) as $c = (s = 0, t = 0)$. Setting the maximum values for numbers (s,t) in (1), we can consider a finite and infinite gratings.

Variation of node coordinates within domain (2), in accordance with coordinate representation (1), will correspondingly affect positions of nodes beyond domain (2), what allows reconstructing the spatial configuration of the grating.

A distance between grating nodes c and c', node c and an arbitrary point in space (x, y, z) looks like (Fig. 1)

$$r_{cc'} = \sqrt{(x_{s'} - x_s)^2 + (y_{t'} - y_t)^2}$$

$$r_c = \sqrt{(x - x_s)^2 + (y - y_t)^2 + z^2}.$$

Centers of small inhomogeneous resonant magnetodielectric spheres with permittivity ε, and permeability μ, and radius α are placed in grating (1) nodes. The permittivity and the permeability of infilling space outside spheres are ε_0 and μ_0, respectively.

Let us present the field in the view $\vec{A}(\vec{r},t) = \vec{A}(\vec{r})e^{i\omega t}$, $\vec{H}(\vec{r},t) = \vec{H}(\vec{r})e^{i\omega t}$.

Assume that outside spheres $\alpha/\lambda' << 1$ and maybe d/λ', $h/\lambda' \sim 1$, whereas inside the sphere a resonant case $a/\lambda_g \sim 1$, where λ' is a wavelength outside the sphere and λ_g is a wavelength within the sphere, is possible.

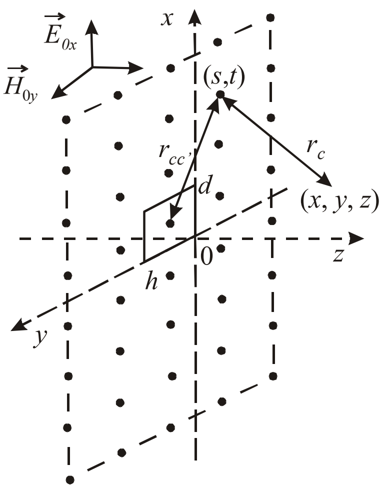

Fig.1. A plane grating and geometry of the problem

A plane electromagnetic wave propagating in the direction of z-axis falls on a plane grating. Confine ourselves by consideration of a case of wave polarization, when vector $\vec{E}_{0x}$ is parallel to axis $0x$ (Fig. 1).

Define the scattered field by the known internal field of scatterers through electric $\vec{\vec{I}}^{e}$ and magnetic $\vec{\vec{I}}^{m}$ Hertz potentials of the plane grating

$$\begin{aligned} \vec{A}_{scatt} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\vec{I}}^{e} - ik\mu_0\left[\nabla, \vec{\vec{I}}^{m}\right], \\ \vec{H}_{scatt} &= \left(\nabla\nabla + k^2\varepsilon_0\mu_0\right)\vec{\vec{I}}^{m} + ik\varepsilon_0\left[\nabla, \vec{\vec{I}}^{e}\right]. \end{aligned} \tag{3}$$

Assume that the fields of the incident wave

$$\vec{A}_{ox}(z,t) = \vec{A}_o e^{i(\omega t - k_1 z)},$$

$$\vec{H}_{oy}(z,t) = \vec{H}_o e^{i(\omega t - k_1 z)}$$

with in spheres of the plane grating have respectively the same values for all the spheres of the grating.

The field of the incident wave with regard to the scattering sphere is presented as an infinite sum of spatial harmonics [3]

$$\begin{aligned} \vec{E}_{0x}(z',t) &= \sum_{m,n} \vec{E}_0^{mn}(\vec{r}',t) \\ \vec{H}_{0y}(z',t) &= \sum_{m,n} \vec{H}_0^{mn}(\vec{r}',t) \end{aligned} \tag{4}$$

Let us write also the internal field in the view of decomposition

$$\begin{aligned} \vec{E}^0(\vec{r}',t) &= \sum_{m,n} \vec{E}^{0mn}(\vec{r}',t), \\ \vec{H}^0(\vec{r}',t) &= \sum_{m,n} \vec{H}^{0mn}(\vec{r}',t), \end{aligned} \tag{5}$$

which must not be considered as the Fourier decomposition.

Then the algebraic equations for multipliers of internal fields $\vec{E}^{0mn}(\vec{r}',t)$, $\vec{H}^{0mn}(\vec{r}',t)$, (5) of an arbitrary sphere of the grating take the form [3]

$$\vec{E}_0^{mn}(\vec{r}',t) = A_\varepsilon^0 \vec{E}^{0mn}(\vec{r}',t) - \sum_{\substack{s \\ (s,t)\neq(s',t')}}\sum_t \frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\frac{\chi_{mn}}{\beta_{mn}}\left\{(\nabla\nabla + k^2\varepsilon_0\mu_0)\left(\frac{\varepsilon_{ef}}{\varepsilon_0}-1\right)\times\right.$$

$$\left.\times\vec{E}^{0mn}(\vec{r}',t) - ik\mu_0\left[\nabla,\left(\frac{\mu_{ef}}{\mu_0}-1\right)\vec{H}^{0mn}(\vec{r}',t)\right]\right\}e^{-i\left[\frac{\pi m}{d}(x_{s'}-x_s)+\frac{\pi n}{h}(y_{t'}-y_t)+\beta_{mn}z_p\right]},$$

$$\vec{H}_0^{mn}(\vec{r}',t) = A_\mu^0 \vec{H}^{0mn}(\vec{r}',t) - \sum_{\substack{s \\ (s,t)\neq(s',t')}}\sum_t \frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\frac{\chi_{mn}}{\beta_{mn}}\left\{(\nabla\nabla + k^2\varepsilon_0\mu_0)\left(\frac{\mu_{ef}}{\mu_0}-1\right)\times\right.$$

$$\left.\times\vec{H}^{0mn}(\vec{r}',t) + ik\varepsilon_0\left[\nabla,\left(\frac{\varepsilon_{ef}}{\varepsilon_0}-1\right)\vec{E}^{0mn}(\vec{r}',t)\right]\right\}e^{-i\left[\frac{\pi m}{d}(x_{s'}-x_s)+\frac{\pi n}{h}(y_{t'}-y_t)+\beta_{mn}z_p\right]}, \quad (6)$$

where

$$A_\varepsilon^0 = \frac{(\varepsilon_{ef}+2\varepsilon_0)+\theta_1^2\varepsilon_{ef}+i\theta_1(\varepsilon_{ef}+2\varepsilon_0)}{3\varepsilon_0 e^{i\theta_1}};\ A_\mu^0 = \frac{(\mu_{ef}+2\mu_0)+\theta_1^2\mu_{ef}+i\theta_1(\mu_{ef}+2\mu_0)}{3\mu_0 e^{i\theta_1}};$$

$$\theta_1^2 = k^2a^2\varepsilon_0\mu_0 .$$

Components $\vec{E}^{0mn}(\vec{r},t)$, $\vec{H}^{0mn}(\vec{r},t)$ of internal fields (5) of singled-out sphere c' of the plane grating are determined from separate algebraic interconsistent systems of algebraic equations made from equations (6), and, as a result, the total internal field of the c'-th sphere is presented as

$$\vec{E}^0(\vec{r}',t) = \sum_{m,n}\left[\frac{1}{\Delta^{mn}}\sum_c\left(\hat{g}_c^{e\tilde{n}'mn}\vec{E}_0^{mn}(\vec{r}',t)+\hat{\beta}_c^{e\tilde{n}'mn}\vec{H}_0^{mn}(\vec{r}',t)\right)\right],$$

$$\vec{H}^0(\vec{r}',t) = \sum_{m,n}\left[\frac{1}{\Delta^{mn}}\sum_c\left(\hat{g}_c^{m\tilde{n}'mn}\vec{E}_0^{mn}(\vec{r}',t)+\hat{\beta}_c^{m\tilde{n}'mn}\vec{H}_0^{mn}(\vec{r}',t)\right)\right], \quad (7)$$

where Δ^{mn} is the determinant of the interconsistent system of algebraic equation (6).

Let us present Hertz potentials of the field scattered by the grating in the form of superposition of Hertz potentials of individual spheres of the grating [3]

$$\vec{\vec{\Gamma}}^e(\vec{r},t) = \frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\left(\frac{\varepsilon_{ef}}{\varepsilon_0}-1\right)\vec{\vec{A}}^0(\vec{r}',t)\sum_{-s}^{s}\sum_{-t}^{t}\left(\sum_{m,n}\frac{\chi_{mn}}{\beta_{mn}}e^{-i\left[\frac{m\pi}{d}(x_s-x)+\frac{n\pi}{h}(y_t-y)+\beta_{mn}z\right]}\right),$$

$$\vec{\vec{\Gamma}}^m(\vec{r},t) = -\frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\left(\frac{\mu_{ef}}{\mu_0}-1\right)\vec{\vec{H}}^0(\vec{r}',t)\sum_{-s}^{s}\sum_{-t}^{t}\left(\sum_{m,n}\frac{\chi_{mn}}{\beta_{mn}}e^{-i\left[\frac{m\pi}{d}(x_s-x)+\frac{n\pi}{h}(y_t-y)+\beta_{mn}z\right]}\right), \quad (8)$$

where [2, 3]

$$\varepsilon_{ef} = \varepsilon\cdot F(ka\sqrt{\varepsilon\mu}),\ \mu_{ef} = \mu\cdot F(ka\sqrt{\varepsilon\mu}),\ k = 2\pi/\lambda,\ k_1^2 = k^2\varepsilon_0\mu_0,$$

$$F(ka\sqrt{\varepsilon\mu}) = \frac{2(\sin ka\sqrt{\varepsilon\mu} - ka\sqrt{\varepsilon\mu}\cos ka\sqrt{\varepsilon\mu})}{(k^2a^2\varepsilon\mu-1)\sin ka\sqrt{\varepsilon\mu}+ka\sqrt{\varepsilon\mu}\cos ka\sqrt{\varepsilon\mu}}, \quad (9)$$

$$\chi_{mn} = \begin{cases} 2\ if & m=0\ or\ n=0, \\ 1\ if & m,n>0 \end{cases} \quad \beta_{mn} = \sqrt{k^2\varepsilon_0\mu_0 - \left(\frac{m\pi}{d}\right)^2 - \left(\frac{n\pi}{h}\right)^2} \quad (m,n=0,1,2,...).$$

Quantities m, n associated with propagating and attenuating spatial harmonics are defined respectively by the following conditions

$$k^2\varepsilon_0\mu_0 > \left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{h}\right)^2,\ k^2\varepsilon_0\mu_0 < \left(\frac{m\pi}{d}\right)^2 + \left(\frac{n\pi}{h}\right)^2.$$

When using (3), (7), (8), the field scattered on the grating is to be found in the form:

$$\hat{E}_{scatt}(\vec{r},t)=\sum_c \frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\sum_{m,n}\frac{\chi_{mn}}{\beta_{mn}}\left\{\left(\frac{\varepsilon_{ef}}{\varepsilon_0}-1\right)\hat{L}^{mn}\vec{E}^0(\vec{r}\,')-ik\mu_0\left(\frac{\mu_{ef}}{\mu_0}-1\right)\hat{P}^{mn}\vec{H}^0(\vec{r}\,')\right\}\times$$

$$\times e^{i\left(\omega t-\left[\frac{m\pi}{d}(x-x_s)+\frac{n\pi}{h}(y-y_t)+\beta_{m,n}z\right]\right)},$$

$$\hat{H}_{scatt}(\vec{r},t)=\sum_c \frac{2\pi}{dhk_1^3}(\sin k_1 a - k_1 a\cos k_1 a)\sum_{m,n}\left\{\left(\frac{\mu_{ef}}{\mu_0}-1\right)\hat{L}^{mn}\vec{H}^0(\vec{r}\,')+ik\varepsilon_0\left(\frac{\varepsilon_{ef}}{\varepsilon_0}-1\right)\hat{P}^{mn}\vec{E}^0(\vec{r}\,')\right\}\times$$

$$\times e^{i\left(\omega t-\left[\frac{m\pi}{d}(x-x_s)+\frac{n\pi}{h}(y-y_t)+\beta_{m,n}z\right]\right)},$$

where $\hat{L}^{mn}$, $\hat{P}^{mn}$ are functional matrices looking like

$$\hat{L}^{mn}=\begin{bmatrix}\left(k^2\varepsilon_0\mu_0-\frac{m^2\pi^2}{d^2}\right) & -\frac{m\pi}{d}\frac{n\pi}{h} & -\beta_{mn}\frac{m\pi}{d}\\ -\frac{m\pi}{d}\frac{n\pi}{h} & \left(k^2\varepsilon_0\mu_0-\frac{n^2\pi^2}{h^2}\right) & -\beta_{mn}\frac{n\pi}{h}\\ -\beta_{mn}\frac{m\pi}{d} & -\beta_{mn}\frac{n\pi}{h} & \left(k^2\varepsilon_0\mu_0-\beta_{mn}^2\right)\end{bmatrix},\quad \hat{P}^{mn}=\begin{bmatrix}0 & i\beta_{mn} & -i\frac{n\pi}{h}\\ -i\beta_{mn} & 0 & i\frac{m\pi}{d}\\ i\frac{n\pi}{h} & -i\frac{m\pi}{d} & 0\end{bmatrix}.$$

The field in an arbitrary point in space takes the form

$$\vec{E}(\vec{r},t)=\vec{E}_{0x}(z,t)+\vec{E}_{scatt}(\vec{r},t),$$

where $\vec{E}_{0x}(z,t)$ is the undisturbed field of the incident wave.

From determinants of the equation systems (6), the resonant conditions are to be found for the case when $a/\lambda_g\sim 1$ within spheres. If ε,μ of grating spheres are real, then it is possible to define the resonant conditions from the expression

$$\det\operatorname{Re}\left\|\alpha_{ij}^{m,n}\right\|=0,$$

when solving it with respect to function $F\left(ka\sqrt{\varepsilon\mu}\right)$ (9), where $\left\|\alpha_{ij}^{m,n}\right\|$ is a basic matrix of the system of equations (6).

Conclusions

In the work, the expressions for the internal fields and fields scattered on grating spheres, which take into account the influence of structural and internal resonances of grating spheres on each other are obtained. This solution can be useful when developing devices for controlling the radiation fields of electromagnetic radiators.

The proposed mathematical models will be of help when both developing new kinds of artificial crystal nanomaterials with resonance properties and studying resonance phenomena in real crystals.

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Харьковский национальный
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