УДК 519.87

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# MARKOVIAN APPROACH TO MAN-MACHINE-ENVIRONMENT SYSTEMS

## Introduction

A Man–Machine- Environment systems (MME) includes such tow subsystems as "human" and "environment" that are of random nature . This means that random phenomena that are taken into account , are subject to certain static patterns , which are not mandatory requirements.

The purpose of this work is to investigate the MME system as a kind of Markovian process . The condition of static stability can be used in decision-making effective mathematical methods in the theory of random processes and , in particular , Markov processes application for the MME this approach is rarer new [1] .

Despite the above-mentioned simplicity and clarity , the practical application of the theory of Markov chains requires knowledge of some basic terms and provisions .

The ergodic chain can be regular or cyclic. Cyclic chains differ from the regular in that process of transition after a certain number of steps (cycles) will return in any state. Regular chains do not have this property. We can give the following classification of Markov processes (Fig. 1):

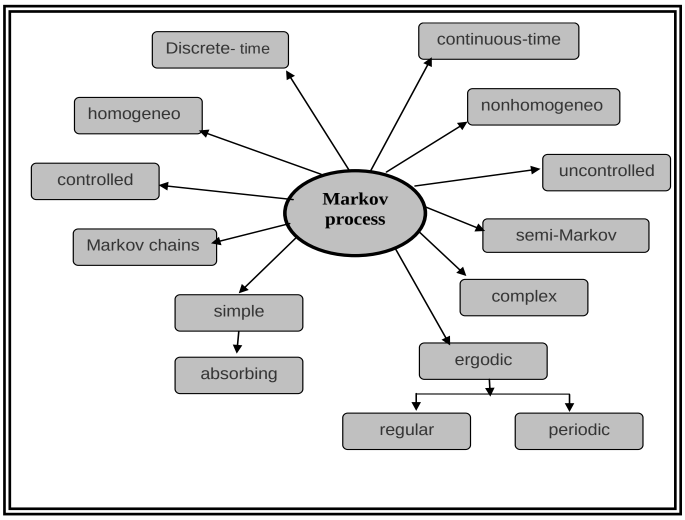



Fig. 1. Classification of Markov processes

## Homogeneous Markov chain as the model of MME-process

The main characteristics of Markov chains are the probabilities

$$P_i(k) = P(S_i(k))(i = 1,..., n; k = 1,2,...)$$

of states $S_i(k)$ at k-th step.

If the transition probabilities do not depend on step k, then Markov chain is called homogeneous. If at least one probability varies with the step k, the chain is called non-homogeneous. The transition probabilities are written in the form of a square matrix of order n. The sum of the elements for each row is 1.

$$p = (p_{ij})_{i,\, j=1}^{n} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}$$

$$\sum_{j=1}^{n} p_{ij} = 1, i = 1, \ldots, n$$

The presence of the arrows in marked graphs with the corresponding transition probabilities from one state to another means that these probabilities are different from zero. Probability of delay $p_{ii}(i = 1,\ldots,n)$ can be obtained as

$$p_{ii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{n} p_{ij} \quad ,(i = 1, \ldots, n)$$

Row vector of probabilities of states $(P_1(0),\ldots, P_n(0))$ at t = 0, is called the vector of initial probability distribution .

**The n-step transition probability**

The probability for transition from state $i$ to state $j$ after $n$ steps is called "The n-step transition probability" , and is denoted by the symbol ( $P_{ij}(n)$) . It is defined the following relationship

$$p_{ij}^{(n)} = P(X_{n+m} = j \mid X_m = i), \quad n \geq 0, \; i, j = 0,1,2,\ldots$$

This matrix $P^{(n)}$ is called the transition probability matrix after step n.

**Chapman-Kolmogorov equations**

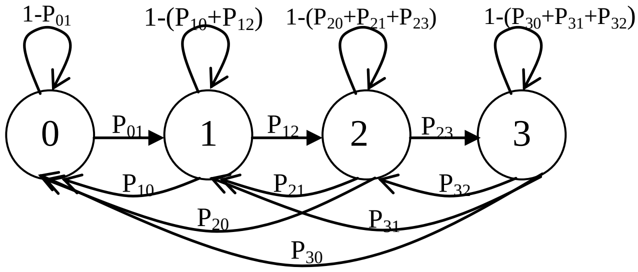
If $\{X_n : n = 0,1,2,\cdots\}$ is the Markov , chain and the number of states m is limited if the transition probability matrix is $P = (p_{ij})$ then :

$$p_{ij}^{(n)} = \sum_{k=1}^{m} p_{ik}^{(r)} \, p_{kj}^{(n-r)}, \quad \forall\, r = 1, 2, \ldots, n-1.$$

Where :

$$p_{ij}^{(n)} = P(\{X_n = j \mid X_0 = i\})$$

Example (1) : for a crew of professional that fits some damage in the system in 3 steps




$$P = \begin{bmatrix} 1-p_{01} & p_{01} & 0 & 0 \\ p_{10} & 1-(p_{01}+p_{12}) & p_{12} & 0 \\ p_{20} & p_{21} & 1-(p_{20}+p_{21}+p_{23}) & p_{23} \\ p_{30} & p_{31} & p_{32} & 1-(p_{30}+p_{31}+p_{32}) \end{bmatrix}$$

Let p01=0.5 , p21=0.5 , p20=0.5 , p30=0.5 , p32=0.5 , p10=0.5 , p12=0.5 , p23=0.5 , p31=0.5.

The eigenvalues of this system we get from the following relation $Det(P-\lambda I)=0$ it gives the characteristic polynomial :

$$\lambda^4 + 0.5\lambda^3 - \lambda^2 - 0.5\lambda = 0$$

The eigenvalues and the appropriate vectors we get by using Mathematica 7:

$\lambda_1 = 1$ The eigenvector is (1,1,1,1)

$\lambda_2 = 0$ The eigenvector is (-1,1,1,1)

$\lambda_3 = -1$ The eigenvector is (-1/3,1,5/3,1)

$\lambda_4 = -0.5$ The eigenvector is (1,-2,1,1)

$$\lim_{n\to\infty} P^n = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

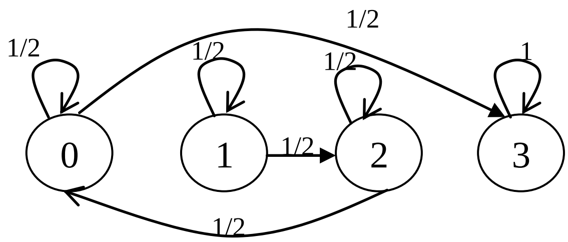
**Absorbing Markov chains**

A state Si of a Markov chain is called absorbing if it is impossible to leave it (i.e., Pii = 1). A Markov chain is absorbing if it has at least one absorbing state, and if from every state it is possible to go to an absorbing state (not necessarily in one step). And a state which is not absorbing is called transient.

So far, we have focused on regular Markov chains for which the transition matrix P is primitive. Because permittivity requires P(i, i) < 1 for every state i, regular chains never get "stuck" in a particular state. However, other Markov chains may have one or more absorbing states. By definition, state i is absorbing when P(i, i) = 1 (and hence P(i, j) = 0 for all j $\neq$ i). In turn, the chain itself is called an absorbing chain when it satisfies two conditions. First, the chain has at least one absorbing state.

Second, it is possible to transition from each non-absorbing state to some absorbing state (perhaps in multiple steps). Consequently, the chain is eventually "absorbed" into one of these states.[4]

Example (2): represent the mortal case happened while fitting the damage of the MME – system




$$P = \begin{bmatrix} 1-p_{01} & 0 & 0 & p_{03} \\ 0 & 1-p_{12} & p_{12} & 0 \\ p_{20} & 0 & 1-p_{20} & 0 \\ 0 & 0 & 0 & p_{33}) \end{bmatrix}$$

Similarly to the example (1) we get , the eigenvalues and the appropriate vectors:

$\lambda_1 = 1$ The eigenvector is (1,1,1,1)

$\lambda_2 = 1/2$ The eigenvector is (0,1,0,0)

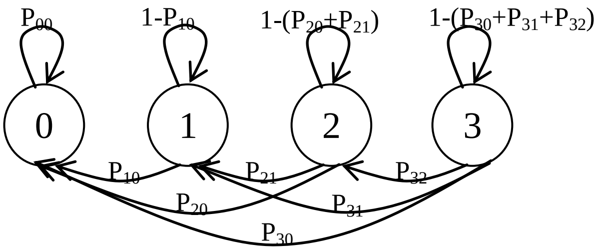
$\lambda_3 = 1/2$ The eigenvector is (0,1,2,0)

$\lambda_4$ = 1/2 The eigenvector is (4,1,2,0)

Then :

$$P^n = \begin{bmatrix} 2^{-n} & 0 & 0 & 1-2^{-n} \\ \frac{n(n-1)}{2^{n+1}} & 2^{-n} & 2^{-n}n & 1-\frac{n^3+n+2}{2^{n+1}} \\ 2^{-n}n & 0 & 2^{-n} & 1-\frac{n+1}{2^m} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example (3): shows the state of operator's health. State "0" is mortal .




We get as before:

$\lambda_1$ = 1 The eigenvector is (1,1,1,1)

$\lambda_2$ =1/4 The eigenvector is (0,0,0,1)

$\lambda_3$ = 1/4 The eigenvector is (0,0,4,1)

$\lambda_4$= 1/4 The eigenvector is (0,16,-12,1)

$$p^n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1-4^{-n} & 4^{-n} & 0 & 0 \\ 1-7(4^{1-n})-4^{-2-n}*(16n-12) & 3(4^{1-n})+4^{-2-n}*(16n-12) & 4^{-n} & 0 \\ 1-\frac{7}{16}(4^{-n}+2n)+\frac{1}{16}(-4(-1+n)n^2-4^{-n}(1+4n)) & \frac{3}{16}(4^{-n}+2n)+\frac{1}{16}(4(-1+n)n^2-4^{-n}(1+4n)) & \frac{1}{4}(4^{-n+2n)} & 0 \end{bmatrix}$$

**Model (1):**

Let the person responsible for the impact (crash) react with the determined sequence of n technological operations, the duration of which is exponentially distributed with parameter μi.

It is natural to consider the operations of an emergency as the state space. S0 is the state of expected trouble-free operation of the system. The Kolmogorov equation for the probability of states in the natural condition of normalization $\Sigma P_i$ (t) ≡ 1 we construct in a standard way. The limit as t → ∞ of the probability Pi exists, they are stationary and do not depend on the initial probability distribution.

When $P'_i$ (t) = 0 and $\Sigma P_i$ = 1, solving the resulting recurrence algebraic equations, we obtain, very similar to the classical formula of Erlang, the limiting state probabilities:

$$P_0 = \frac{1}{(1+\sum \frac{\lambda}{\mu_i})}; P_i = \frac{\frac{\lambda}{\mu_i}}{(1+\sum_{i=1}^{n} \frac{\lambda}{\mu_i})}, i = 1..n$$

**Model (2):**

With all the assumptions of the previous model, time and quality of operations for emergency response depends on the human operator’s health, which, in turn, depends on the state of the system (and the harmful effects of stress). We assume that the operator can be in two states of health, and call them "healthy" and "sick", assuming the probability of recovery of health in the process of liquidation of the accident is zero, and the probability of being sick during the i-th operation is bi. After completing all the work (and before the time of the next emergency comes) the operator's health is restored, or he is replaced. Then the transition probabilities for each pair of neighboring states of the operator is $b_i\mu_i$ and $(1-b_i)\ \mu_i$ . The state of the system, the intensity of rehabilitation and the probabilities regarded to incomplete performance of the operator, respectively, are $i_b$, $\mu_b$ and $P_{ib}$.

From the recurrence relations obtained in the limiting case of the corresponding equations of Kolmogorov [5], with the notation:

$$B_k = \prod_{j=1}^{k-1}(1-b_j); \qquad A_k = 1 - B_k; \qquad A_0 = A_1 = 0$$

$$B_0 = B_{1=1}; \qquad \sum 1 = \sum_0^n B_k / \mu_k, \sum 2 = \sum_0^n A_k / \mu_k \quad ,$$

we get with k = 2, ..., n :

$$P_k = \frac{B_k}{\mu_k(\Sigma_1 + \Sigma_2)}, \ P_{k,b} = \frac{A_k}{\mu_k^b(\Sigma_1 + \Sigma_2)}$$

$$P_0 = \frac{B_{n+1}}{\lambda(\Sigma_1 + \Sigma_2)}, P_{0,b} = \frac{A_{n+1}}{\lambda(\Sigma_1 + \Sigma_2)}, \ P_0^* = \frac{1}{\lambda(\Sigma_1 + \Sigma_2)}$$

The probabilities $P^* = P_i P_{ib}$, $1 - P^*_0$ and $P_{0b}$ are usually of practical interest.

In conclusion, we note that the assumption under which the queuing system adequately simulates our system is that the flow of events is stationary. For very small $\lambda << 1$ it is valid for the considered flow.

However, from the exponential distribution of lengths of intervals between accidents follows that short intervals are most likely. Thus, the QS model is best suited for rescue teams, for which the disaster is a "steady state". Experiments in MathCAD for the case n= 3, 5, 10 (the number of operations in the processing chain to address the accident) and $\lambda$= 0.3, 0.1, 0.05, $\mu_i$ = 0.8 for P(S0) = 1 has confirmed that the settling time is $t \cong 5$. In this case, an average time is less than the time of the system's operation ($1 / \lambda + n / \mu$ ).

The above proposed model is naturally extended to the case of other different disasters (Erlang flow and absorbing states), but now it becomes clear that the accident, and even more disasters have different distributions, i.e., power distribution. This leads to more sophisticated than QS mathematical models, which do not fit the classical theorem by Khintchin [5] on the convergence to a simple flow.

**Reference:** 1. *Daniel P. Heyman and Matthew J. Sobel*. Stochastic Models in Operations Research, 2004.
2. *B. V. Ramana*. Higher Engineering Mathematics, 2007**.** 3.*Yu. Kabanova ,R. Liptser and J. Stoyanov* From stochastic calculus to Mathematical Finance, 2006. 4. *Warren Ewens* Mathematical Population Genetics // Second edition, 2004. – P. 104-109. 5. *Хинчин, А.Я.* Работы по математической теории массового обслуживания – М. : ГИ Физ-мат.лит., 1963. – 235с.

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*Поступила в редколлегию 05.06.2012*

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