УДК 621.372(075)

O.I. KHARCHENKO, V.M. KARTASHOV

VOLTERRA TRANSFER FUNCTIONS IN ANALYSIS OF THE STOCHASTIC FILTER DRIVEN BY HARMONIC PLUS GAUSSIAN NOISE INPUT

Introduction in Volterra series analysis

In communication systems often it is necessary to deal with the devices executing non-linear conversions. Volterra series are usually used for calculation of such devices. Wide class of communication systems can be computed with the help Volterra series. Wiener introduced Volterra series into nonlinear circuit analysis [1].

Here we apply Volterra-series-type analyses to systems driven by harmonic and Gaussian input.

Volterra series describe the output of a nonlinear system in degrees of input $x(t)$. A substantial number of the communication system can be represented as Volterra series. The series for typical system can be writing as [2]

$$y(t)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}du_1...\int_{-\infty}^{\infty}du_n g_n\left(u_1,...,u_n\right)\prod_{r=1}^{n}x\left(t-u_r\right), \tag{1}$$

where $y(t)$ is the output, $x(t)$ - the input and the kernels $g_n\left(u_1,...,u_n\right)$ describe the nonlinear system. The first-order kernel $g_1(u_1)$ is simply the familiar impulse response of linear network. The higher order kernels of higher order impulse characterize the various orders of nonlinearity.

The coefficient $1/n!$ inserted A. Bedrosian and D. Rice [2] simplifies many of equations.

Further analysis require using the n-fold Fourier transform which has the form [2, 3]

$$G_n\left(f_1,...,f_n\right)=\int_{-\infty}^{\infty}du_1...\int_{-\infty}^{\infty}du_n g_n\left(u_1,...,u_n\right)\exp\left[-j\left(f_1u_1+...f_nu_n\right)\right] \tag{2}$$

Here G_0 is identically zero because the Volterra series starts with $n=1$, and $G_1\left(f_1\right)$ is the transfer function of linear circuit. For linear systems, the possible output frequencies are the same as the frequencies in the input. For nonlinear systems, however, the relationship between the input and output frequencies is more complicated [4, 5].

Thus the transform of the nth-order Volterra kernel is seen to be analogous to an nth-order Volterra transfer function. In many cases G_n can be obtained without first computing g_n.

The complete formulas are infinite series. Fortunately, in the study of communication system it is often possible to neglect terms of the Volterra series of order higher than the second or third. They are usually used because of fast increase in complexity [2, 3]. The n-fold Fourier transform considerably simplifies the solution of a large number of problems.

To calculate the transfer functions, we use the harmonic input method [2]. This method relies on the fact that a harmonic input must result in a harmonic output when (1) holds. System specified by the nonlinear differential equation are considered in [2, 3]

$$F(d/dt)y+\sum_{l=2}^{\infty}a_l y^l=x(t), \tag{3}$$

with the condition that system causally ($y(t)$ vanish identically when $x(t)$ does). It is assumed that one and only one such solution exists (it is proved in [3]) and the system is stable. $F(d/dt)$ is a polynomial in d/dt, and the coefficients in $F(d/dt)$ and the coefficients a_l are independent of t, x and y.

The Volterra transfer functions for (3) can be written as [2]

$$G_n(f_1,...,f_n) = -\frac{\sum_{l=2}^{n} a_l G_n^{(l)}(f_1,...,f_n)}{F(j\omega_1 + ... + j\omega_n)}. \quad (4)$$

The last equation is recurrence relation because $G_n^{(l)}$ is given by

$$G_n^{(l)}(f_1,...,f_n) = l! \sum_{(\nu;l,n)} \sum_{l=2}^{n} a_l G_{\nu 1}(f_1,...,f_{\nu 1}) G_{\nu 2}(f_{\nu 1+1},...,f_{\nu 1+\nu 2})...G_{\nu l}(f_\mu,...,f_n),$$

for the n-fold Fourier transform of the n-th kernel in the Volterra series for $[y(t)]^l$, l being a positive integer, and $1 \le l \le n$. $G_n^{(l)}(f_1,...,f_n)$ is zero for $l > n$ and $G_n^{(n)}(f_1,...,f_n)$ is equal to $n! G_1(f_1) G_1(f_2)...G_1(f_n)$.

Phenomenon of Stochastic Resonance

It is usually considered that noise in a system is a negative factor and the fight against noise is one of actual problems of radio engineering systems. Low-noise devices and methods of noise reduction are developed, noiseproof codes, digital communication, signals with the necessary correlation properties are created. However, research conducted recently in the field of theoretical and experimental physics has shown that in some cases an input weak signal can be amplified and optimized with the assistance of noise [6, 7]. The system output integral characteristics, such as the spectral power amplification, the signal-to-noise ratio (SNR) have a well-marked maximum at a certain optimal noise level.

The notion of stochastic resonance (SR) determines a group of phenomena where in the response of a nonlinear system to a weak input signal can be significantly increased by appropriate tuning of the noise intensity. SR refers to a generic physical phenomenon typical for nonlinear systems.

A weak input signal significantly increases with increasing intensity of noise and reaches its maximum at a certain noise level in nonlinear systems in which SR occurs.

SR equation has the form [6, 7]

$$\frac{dy}{dt} = ay(t) - by(t)^3 + x(t), \quad (5)$$

where a and b are positive, $x(t) = s(t) + n(t)$, $s(t) = A\sin(2\pi f_0 t + \varphi)$ is the driving signal, $n(t)$ is the input Gaussian noise.

Using equation (4) we can find Volterra transfer function for SR equation. Volterra transfer functions for $y(t)$ are given in table 1 for the general case (the equation 3) and for SR equation.

Nonlinear stochastic filtration of the Minimum Shift Keying waveform

It is used Gaussian Minimum Shift Keying. (GMSK) in the GSM standard. It has advantages of being able to carry digital modulation while still using the spectrum efficiently. In view of the efficient use of the spectrum in this way, GMSK modulation has been used in a number of radio communications applications [8].

GMSK modulation is based on MSK, which is itself a form of continuous-phase frequency-shift keying. One of the problems with standard forms is that sidebands extend out from the carrier. To overcome this, MSK and its derivative GMSK can be used.

The pulse stream is divided into an in-pulse stream $d_L(t)$ (even bits) and a quadrature stream $d_Q(t)$ (odd bits). The MSK waveform can be expressed as [8]

$$s(t)= s(t) = d_L(t)\cos\frac{\pi t}{2T}\cos 2\pi f_0 t + d_Q(t)\sin\frac{\pi t}{2T}\sin 2\pi f_0 t,$$

where T - bit time slot; f_0 - the carrier.

We will define the MSK waveform for the pulse stream: -1;1; 1;-1; -1;1;-1;1. Fig.1 illustrated equation (6) tor this waveform. Fig. 1a and c show the sinusoidal weighting of the I - and Q-channel pulses.

Fig. 1b and d illustrate the modulation of the orthogonal components $\cos\omega_0 t$ and $\sin\omega_0 t$ respectively. Fig. 1e illustrated the summation of the orthogonal components from Fig.1b and d.

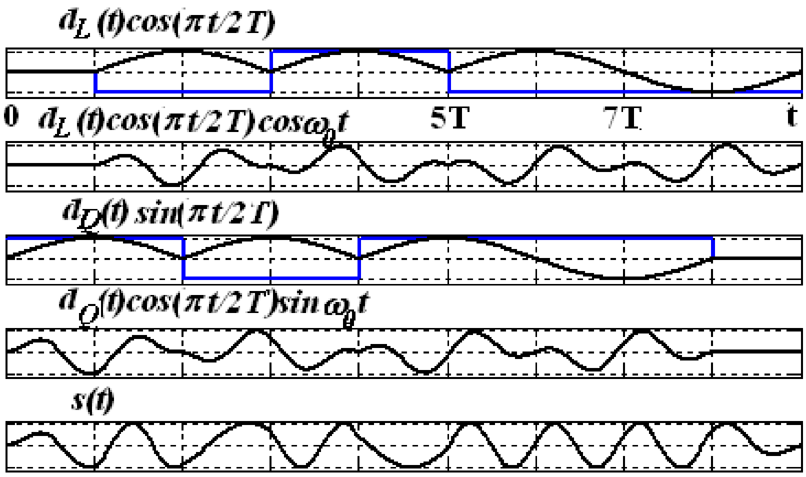

Fig. 1. Example of Minimum shift keying: *a* - Modified I bit stream; *b* - I bit stream times carrier; *c* - Modified Q bit stream; *d* - Q bit stream times carrier; *e* - MSK waveform

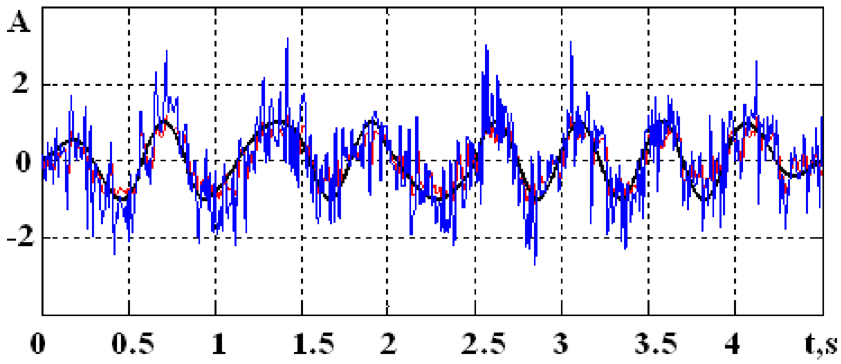

Fig. 2. Standing out of signal from additive mixture of signal and Gaussian noise (input signal – black , additive mixture of signal and Gaussian noise – blue, output signal – red)

It is seen that as a result of processing in accordance with expression (5) can significantly reduce the noise component fluctuations. Dispersion of input noise is equal to 0.7. Dispersion of the output signal is equal to 0.4. Thus the stochastic filter provide effective noise suppression in the communication systems with MSK modulation.

Stochastic Filter Driven by Harmonic plus Gaussian Input

In papers [9, 10] Gaussian random process or sinusoidal passing through the non-linear filter having the effect of a stochastic resonance is researched. Expressions for the output harmonics and output power spectrum were received.

Consider an input signal in form $x(t) = A\cos\omega t + n(t)$, where $n(t)$ is a zero-mean stationary Gaussian process with two-sided power spectrum $W_I(t)$.

The ensemble average of $x(t)$ is $\langle x(t) \rangle = A\cos\omega t$. Similarly, the ensemble average of $y(t)$ consist of a sum of sinusoidal harmonics of $\cos\omega t$.

The leading terms for the power spectrum of $y(t)$ are shown in Table 1. In the first column leading terms of the output power spectrum are given according to [2]. Using this terms and Volterra transfer functions for SR (Table 1), obtain leading terms of the output power spectrum for SR (column 2).

The spikes in $W_Y(f)$ due to sine waves increases gradually with an increase in the noise spectral density. Figure 3 represents first harmonic power spectral densities as a function of the f_0 for several values of the noise spectral density. First harmonic power spectral densities decrease with an increase in the input signal frequency and it decreases gradually on low frequencies.

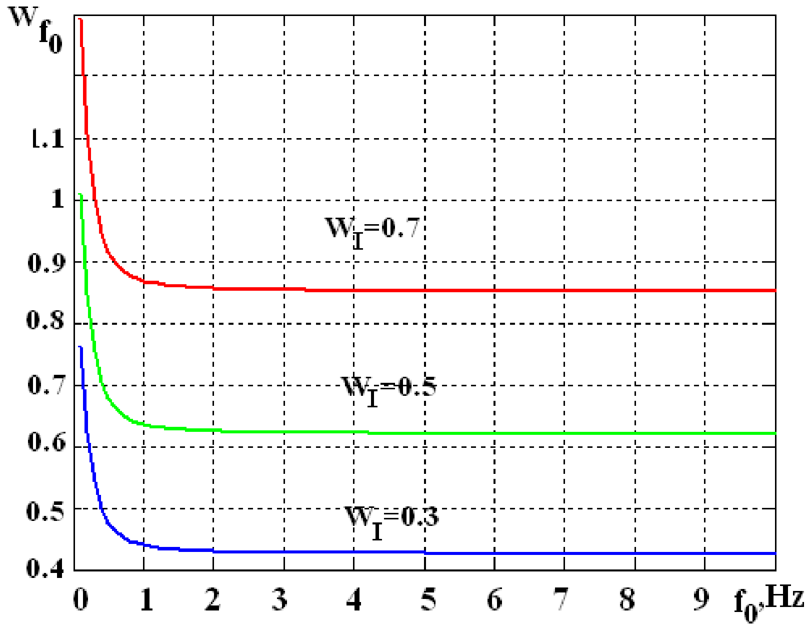

Fig. 3. First harmonic power spectral densities of various noise power spectrum

Considering spectral density of third harmonic power, we see what in case of $A < 1$ it decrease rapidly. Third harmonic belong to interfering signals therefore it shall be small. $A < 1$ is typical characteristic of the SR phenomenon.

Conclusions

The Volterra series is a powerful tool that can be used to describe a wide class of non-linear systems. The Volterra series is widely employed to represent the input-output relationship of non-linear systems. Volterra transfer functions plays an important role in this analysis.

Leading terms of the SR output power spectrum is obtained using Volterra transfer functions.

Results of applying Volterra series analysis to systems with SR effect driven by harmonic plus Gaussian noise input showed what:

- first harmonic power spectral densities decrease with an increase in the input signal frequency and it decreases gradually on low frequencies;
- third harmonic power spectral densities decrease rapidly in case of $A < 1$.

Numerical calculation of a MSK waveform is shows that SR filtration significantly reduce the noise component fluctuations. The obtained results are planned to be further used for the designing and experimental modeling of a digital stochastic filter.

Table 1

	Volterra transfer functions for eq.(3) [2]	Volterra transfer functions for eq. (5)
$G_1(f_1)$	$1/F(j\omega_1)$	$\dfrac{1}{-a+j\omega_1}$
$G_2(f_1,f_2)$	$-2a_2G_1(f_1)G_1(f_2)/F(j\omega_1+j\omega_2)$	0
$G_3(f_1,f_2,f_3)$	$-\dfrac{2a_2\sum\limits_3^{'}G_1(f_1)G_2(f_2,f_3)+6a_3G_1(f_1)G_1(f_2)G_1(f_3)}{F(j\omega_1+j\omega_2+j\omega_3)}$	$\dfrac{-6b}{(-a+j\omega_1)(-a+j\omega_2)(-a+j\omega_3)(-a+j\omega_1+j\omega_2+j\omega_3)}$

Table 2

Leading terms of the output power spectrum $W_Y(f)$ [2]	Leading terms of the output power spectrum for SR
Spikes due to sine waves	
$\delta(f-f_0)\times$ $\left\lvert \frac{A}{2}G_1(f_0)+\frac{A^3}{16}G_3(f_0,f_0,-f_0)+\frac{A}{4}\int\limits_{-\infty}^{\infty}df_1W_1(f_1)G_3(f_1,f_{-1},f_0)\right\rvert^2$	if $a>0$ $\delta(f-f_0)A^2\frac{[2(a^2+\omega_0^2)\{2a^2+3bW_I\}+3A^2ba]^2+16a^2\omega_0^2(a^2+\omega_0^2)^2}{64a^2(a^2+\omega_0^2)^4}$ if $a<0$ $\delta(f-f_0)A^2\frac{[2(a^2+\omega_0^2)\{2a^2-3bW_I\}+3A^2ba]^2+16a^2\omega_0^2(a^2+\omega_0^2)^2}{64a^2(a^2+\omega_0^2)^4}$
$\delta(f-3f_0)\left\lvert\frac{A^3}{48}G_3(f_0,f_0,f_0)\right\rvert^2$	$\delta(f-3f_0)\frac{A^6b^2}{64(a^2+\omega_0^2)^3(a^2+9\omega_0^2)}$
{terms with $-\omega_0$, $-f_0$ for ω_0, f_0 in $e^{jk\omega_0 t}[\ldots]$, $k=1,2,\ldots$}, where $f_0=\omega_0/2\pi$	
$\delta(f+f_0)\times$ $\left\lvert \frac{A}{2}G_1(f_0)+\frac{A^3}{16}G_3(f_0,f_0,-f_0)+\frac{A}{4}\int\limits_{-\infty}^{\infty}df_1W_1(f_1)G_3(f_1,f_{-1},f_0)\right\rvert^2$	if $a>0$ $\delta(f+f_0)A^2\frac{[2(a^2+\omega_0^2)\{2a^2+3bW_I\}+3A^2ba]^2+16a^2\omega_0^2(a^2+\omega_0^2)^2}{16a^2(a^2+\omega_0^2)^4}$ if $a<0$ $\delta(f+f_0)A^2\frac{[2(a^2+\omega_0^2)\{2a^2-3bW_I\}+3A^2ba]^2+16a^2\omega_0^2(a^2+\omega_0^2)^2}{16a^2(a^2+\omega_0^2)^4}$
$\delta(f+3f_0)\left\lvert\frac{A^3}{48}G_3(-f_0,-f_0,-f_0)\right\rvert^2$	$\delta(f+3f_0)\frac{A^6b^2}{64(a^2+\omega_0^2)^3(a^2+9\omega_0^2)}$

Table 2 continuation

$W(f)\left\vert G_1(f)+\frac{A^2}{4}G_3(f_0,-f_0,f)+ \right.$ $\left. +\frac{1}{2}\int\limits_{-\infty}^{\infty}df_1 W(f_1)G_3(f_1,-f_1,f)\right\vert^2$	if $a>0$ $W_I\frac{[(2a^2+3bW_I)(a^2+\omega_0^2)+3A^2ab]^2+4a^2\omega^2(a^2+\omega_0^2)^2}{4a^2(a^2+\omega_0^2)^2(a^2+\omega^2)^2}$
	if $a<0$ $W_x\frac{[(-2a^2+3bW_x)(a^2+\omega_0^2)-3A^2ab]^2+4a^2\omega^2(a^2+\omega_0^2)^2}{4a^2(a^2+\omega_0^2)^2(a^2+\omega^2)^2}$
$W(f-3f_0)\left\vert\frac{A^2}{8}G_3(f_0,f_0,f-2f_0)\right\vert^2$	$\frac{9A^4b^2W_I}{16(a^2+\omega_0^2)^2(a^2+[\omega-2\omega_0]^2)(a^2+\omega^2)}$
term with $-f_0$ for f_0 in $W_I(f-kf_0)\vert...\vert^2, k=1,2,..$	
$W(f+3f_0)\left\vert\frac{A^2}{8}G_3(-f_0,-f_0,f+2f_0)\right\vert^2$	$\frac{9A^4b^2W_I}{16(a^2+\omega_0^2)^2(a^2+[\omega+2\omega_0]^2)(a^2+\omega^2)}$
$\frac{1}{2!}\int\limits_{-\infty}^{\infty}df_1 W(f_1)W(f-f_1-f_0)\times$ $\times\left\vert\frac{A}{2}G_3(f_1,f_0,f-f_1-f_0)\right\vert^2$	if $a>0$ $\frac{9A^2b^2W_I}{2a(a^2+\omega_0^2)(a^2+\omega^2)(4a^2+(\omega-\omega_0)^2)}$
	if $a<0$ $-\frac{9A^2b^2W_I}{2a(a^2+\omega_0^2)(a^2+\omega^2)(4a^2+(\omega-\omega_0)^2)}$

Table 2 ending

$$\frac{1}{2!}\int\limits_{-\infty}^{\infty} df_1 W(f_1)W(f-f_1+f_0)\times$$ $$\times\left\|\frac{A}{2}G_3(f_1,-f_0,f-f_1+f_0)\right\|^2$$	if $a>0$ $$\frac{9A^2b^2W_I}{2a(a^2+\omega_0^2)(a^2+\omega^2)(4a^2+(\omega+\omega_0)^2)}$$
	if $a<0$ $$-\frac{9A^2b^2W_I}{2a(a^2+\omega_0^2)(a^2+\omega^2)(4a^2+(\omega+\omega_0)^2)}$$
$$\frac{1}{3!}\int\limits_{-\infty}^{\infty} df_1 \int\limits_{-\infty}^{\infty} df_2 W(f_1)W(f_2)W(f-f_1-f_2)\times$$ $$\times\left\|G_3(f_1,f_2,f-f_1-f_2)\right\|^2$$	$$\frac{9b^2W_I^3}{2a^2(a^2+\omega^2)(\omega^2+9a^2)}$$

References

1. Wiener N. Nonlinear Problems in Random Theory. Cambridge, Mass.Technology Press; and New York Wiley, 1958.

2. Bedrosian E., Rice S. The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs // IEEE. 1971. Vol. 59, No 12. P.58–82.

3. Pupkov K. A., Kapalin V. I., Yushchenko A.S. Funktsional'nyye ryady v teorii nelineynykh sistem. Moskva : Nauka, 1978. 448p. (in Rus.).

4. Zernov N.V., Karpov V.G. Teoriya radiotekhnicheskikh tsepey. Moskva ; Leningrad : Energiya, 1965. 816 p. (in Rus.).

5. Voloshchuk Yu. I. Pidruchnik dla studentiv vich. navch. zakladiv. Kharkiv : TOV “Kompania CMIT”, 2005. T.3. 228 p. (in Russian).

6. Anishchenko V. S., Neiman A. B., Moss F., Schimansky-Geier L. Stochastic resonance: noise-enhanced order // Uspekhi Fizicheskikh Nauk, Russian Academy of Sciences. 1999. 42(1)7-36. P.7-34.

7. Xiaofei Zhang, Niaoqing Hu, Zhe Cheng. Stochastic resonance in multi-scale bistable array // Physics Letter A 377. 2013. P. 981-984.

8. Sklar B. Digital Communication. Fundamentals and Applications, Second Edition. Prentice Hall PTR, 2003. 1099 p.

9. Kharchenko O., Lonin Yu.F., Ponomaryov A.G.. Frequency analysis of the stochastic filtering using transfer function. Part I. Sinusoidal input // Problems of atomic lposcience and technology. 2018. № 4: Series “Plasma Physics”. Issue 116. P.249-252.

10. Kharchenko O. Ispol'zovaniye apparata peredatochnykh funktsiy Vol'tera v reshenii zadachi stokhasticheskoi filtracii s vhodnym signalom v vide belogo Gaussova shuma [Using of Volterr’s Transfer Functions in Solving the Problem of Stochastic Filtration with Input Signal in the Form of White Gaussian Noise] // Visnik NTUU KPI Seriia Radiotekhnika Radioaparatobuduvannia. 2018. Iss. 73. P. 11 – 16. (in Rus.).

Kharkiv National University of Radio Electronics

Received 05.02.2019